International Journal of Civil, Structural, Environmental and Infrastructure Engineering Research and Development (IJCSEIERD)
ISSN(P): 2249-6866; ISSN(E): 2249-7978
Vol. 4, Issue 2, Apr 2014, 59-64

POST BUCKLING ANALYSIS OF SANDWICH BEAMS WITH FUNCTIONALLY GRADED FACES USING A CONSISTENT HIGHER ORDER THEORY

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ABSTRACT

In this project work, a new consistent higher order theory has been developed for the buckling and post buckling analysis of sandwich beams having functionally graded faces. The model combines an approximation for the displacement fields with third order in the thickness direction. The governing equilibrium equations of motion and the related boundary conditions have been derived. The parametric study of the beam behaviors under different situations is performed and the important influencing parameters are identified.

KEYWORDS: Buckling, Post Buckling, Sandwich Beam, Shear Deformation Theory, Functionally Graded Faces

INTRODUCTION

Functionally graded materials (FGMs), a new generation of composites proposed in the earlier 1980s are achieved by two or more constituent phases with a continuously variable composition along the thickness direction. It has potential application in different field of engineering, especially in defense and aerospace. FGMs have drawn considerable attention as they are made from a mixture of metals and ceramics and are further characterized by a smooth and continuous change of the mechanical properties from one surface to another [1]. Some research has been carried out to analyze the buckling and post-buckling response of structures made of FGMs. But more effort in the research is required for more accurate analysis to explore properties and response under different types of environment and different type of configuration. Yaghoobi and Torabi [1] studied post-buckling and nonlinear vibration analysis of geometrically imperfect beams made of FGMs resting on nonlinear elastic foundation subjected to axial force.

Loja et al [2] studied the static and free vibration behavior of functionally graded sandwich plate type structures, using B-spline finite strip element models based on different shear deformation theories. The effective properties of functionally graded materials are estimated according to Mori–Tanaka homogenization scheme. Simsek and Reddy [3] proposed a unified higher order beam theory which contains various beam theories as special cases for buckling of a functionally graded (FG) micro beam embedded in elastic Pasternak medium based on the modified couple stress theory (MCST). This non-classical micro beam model incorporates the material length scale parameter which can capture the size effect. The non-classical beam model reduces to the classical beam model when the material length scale parameter is set to zero. Li and Batra [4] derived analytical relations between the critical buckling load of a FGM Timoshenko beam and that of the corresponding homogeneous Euler–Bernoulli beam subjected to axial compressive load for clamped–clamped (C–C), simply supported–simply supported (S–S) and clamped–free (C–F) edges. Simsek et al [5] developed a micro scale functionally graded Timoshenko beam model for the static bending analysis based on MCST. They assumed the material properties of the FG micro beams to vary in the thickness direction and are estimated through the Mori–Tanaka homogenization technique and the classical rule of mixture.

Simsek [6] performed non-linear dynamic analysis of a FG beam with pinned–pinned supports due to a moving harmonic load by using Timoshenko beam theory with the von-Karman's non-linear strain–displacement relationships. Huang and Li [7] performed an analysis of the stability of circular cylindrical columns/beams composed of FGM is made where shear deformation is taken into account. The coupled governing equations for the deflection and rotation are derived based on the traction-free surface condition and a single governing equation is further obtained. Singh and Li [8] presented a mathematical model to compute the buckling loads of uniform and non-uniform functionally graded columns in the axial direction. The columns with spatial variation of flexural stiffness, due to material grading and/or non-uniform shape, are approximated by an equivalent column with piecewise constant geometrical and material properties. Kang and Li [9] investigated the mechanical behaviors of a non-linear FGM cantilever beam subjected to an end force are investigated by using large and small deformation theories. Young's modulus is assumed to be depth-dependent. Yang and Chen [10] presented a theoretical investigation in free vibration and elastic buckling of beams made of FGMs containing open edge cracks by using Bernoulli–Euler beam theory and the rotational spring model. They assumed that the material properties vary along the beam thickness only according to exponential distributions.

METHODOLOGY

A new efficient model has been developed for the buckling and post buckling analysis of sandwich beams consisting of functionally graded faces. The model approximates for the axial displacement fields with third order assumption. The model approximates for the materials properties in the thickness direction. The governing equilibrium equations of motion have been derived by using Hamilton's Principle. Analytical solutions are obtained for the beam for buckling and post buckling response under the loads. Parametric study is performed for the present theory

Consider a sandwich beam with functionally graded faces as shown in the figure (Figure 1) having length L, width b, and total constant thickness h. The following distances are taken from the mid surface of the beam:

Distance of bottom fibre of the bottom face = h_1

Distance of bottom fibre of the core = h_2

Distance of top fibre of the core = h_3

Distance of top fibre of the top face = h_4

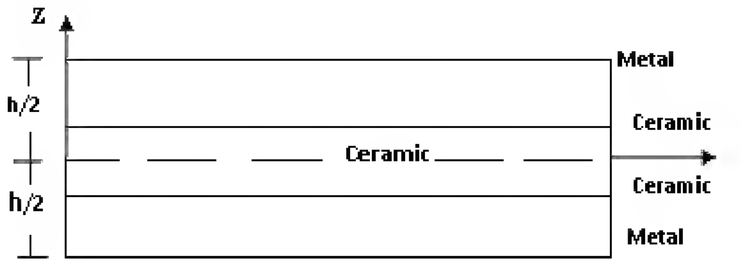

Figure 1: Sandwich Beam with FGM Faces

The constituent materials are taken to be ceramic and metal and its volume fractions of the ceramic and metal follows the power law [11] as:

$$V_c = \left(\frac{z-h_1}{h_2-h_1}\right)^k \; for\ bottom\ face$$

$$V_c = 1 \, for \; core$$

$$V_c = \left(\frac{z-h_4}{h_3-h_2}\right)^k \, for \; bottom \; face$$

$$V_m = 1 - V_c \quad (1)$$

The subscripts m and c in the above equation mean to the metal and ceramic constituents, respectively and z is the coordinate in the thickness direction of the beam. The superscript k is the volume fraction index that varies greater than or equal to zero. For k = 1, the variation is linear for both ceramic and metal and for k = 0, the beam is fully ceramic.

The mechanical properties of FGM beam here are determined as per the volume fraction of the ceramic and metal assumed from the power law. The modulus of elasticity E and density of material ρ assumed in the thickness direction z based on Voigt's rule [11]; and Poisson's ratio ν is assumed to be constant [12] as expressed here

$$E = E(z) = E_c V_c + E_m \, (1 - V_c)$$

$$P = P(z) = P_c V_c + P_m \, (1 - V_c)$$

$$\nu_{(z)} = \nu_0 \quad (2)$$

The displacement field is assumed from the Reddy's third order shear deformation theory [13]

$$u(x,z) = u_0 + z\left[\emptyset_0 - kz^2(\emptyset_0 + \frac{\partial \omega}{\partial x}\right]$$

$$w(x,z) = w_0$$

$$k = \frac{4}{3h^2} \quad (3)$$

In the above equations, u and w are the displacement components in the x and z directions; φ_0 is the rotation of the transverse normal about y axis; the generalized displacements are u_0, w_0, φ_0.

The strain displacement relations are assumed as follows

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial z})^2$$

$$\nu_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \quad (4)$$

The constitutive equations are taken in the formulation as following

$$\sigma_x = Q_{11}(z)\varepsilon_x$$

$$\tau_{zx} = Q_{55}(z)\gamma_{xz} \quad (5)$$

Where

$$Q_{11}(z) = \frac{E(z)}{(1-\nu^2)}$$

$$Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \quad (6)$$

The potential energy (P) and work done (W) [14] by the load can be written as

$$U = \frac{1}{2} \int_V (\sigma_x \epsilon_{xz} + \tau_{zx} v_{xz})\, dv \tag{7}$$

$$W = \frac{1}{2} \int_0^L P(\frac{\partial w}{\partial x})^2\, dx \tag{8}$$

Now, the equilibrium equations are derived with the help the Hamilton's Principle [15]

$$\delta \int_0^t (T - U + W)\, dt = 0 \tag{9}$$

The equilibrium equations are derived as follows:

$$2Pw_{,xx} + 6bk[D_{55}\, \emptyset_{,x} - 3kF_{55}(\emptyset_{,x} + w_{,xx})] +$$

$$2b[A_{11}\, u_{,xxx} + B_{11}\emptyset_{,xxx} - kE_{11}(\emptyset_{,xxx} + w_{,xxxx})] = 0$$

$$A_{11}\, u_{,xx} + B_{11}\emptyset_{,xx} - kD_{11}(\emptyset_{,xx} + w_{,xxx}) = 0$$

$$A_{55}\emptyset - 3kD_{55}(\emptyset + w_{,x}) + 9k^2F_{55}(\emptyset + w_{,x}) = 0 \tag{10}$$

Where

$$A_{11}, B_{11}, D_{11}, E_{11} = \int_{-[\frac{h3}{2}+(h1-h2)]}^{[\frac{h3}{2}+(h4-h3)]} (1, z, z^2, z^3) Q_{11}\, dz$$

$$A_{55}, D_{55}, F_{55} = \int_{-[\frac{h3}{2}+(h1-h2)]}^{\frac{h3}{2}+(h4-h3)]} (1, z^2, z^4) Q_{55}\, dz \tag{11}$$

The following displacement fields [16] are assumed for the first buckling mode for simply supported beam:

$$w(x) = \alpha \sin(\frac{\pi x}{L})$$

$$\emptyset(x) = \beta \cos(\frac{\pi x}{L}) \tag{12}$$

For clamped-clamped boundary condition, the following displacement [18] field is taken for analysis:

$$w(x) = \cos\left(\frac{2\pi x}{L}\right) - 1$$

$$\emptyset(x) = \mathrm{scos}\left(\frac{2\pi x}{L}\right) - 1 \tag{13}$$

For simply supported beam, the critical buckling load calculated as

$$P = \left[(bkD_{11} - bkE_{11})\frac{\pi^2}{L^2} + 9bk^2F_{55}\right] - [\frac{(qk^2F_{55} - 3KD_{55})\{(18bk^2F_{55} - 6bkD_{55}) + \frac{\pi^2}{L^2}(2bkD_{11} - 2bkE_{11})\}}{A_{55} - 3kD_{55} + 9k^2F_{55}}]$$

RESULTS

We analyze a sandwich beam with functionally graded faces having alumina (ceramic) and aluminium (metal) as shown in the figure (Figure 1) with Ec = 380 GPa, Em = 70 GPa, ν = 0.23 [17]. The top and bottom faces of the sandwich beam are taken 0.1 h each. The analysis is performed on critical buckling load with the effect of aspect ratio of the beam on it and post buckling response. It is observed that shear plays a major role on critical buckling load and post buckling response as well. In Figure 2, the post L/h versus critical buckling load is plotted for simply supported beam; whereas variation of buckling load is plotted against length to thickness ratio of for clamped-clamped beam in Figure 3.

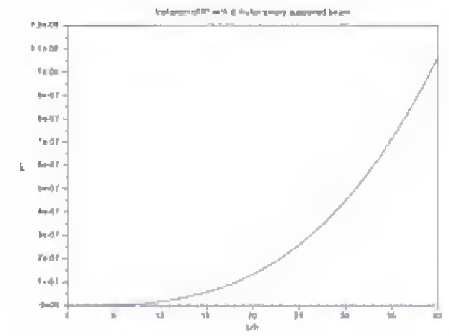

Figure 2: Variation of Buckling Load with L/h for Simply Supported Beam

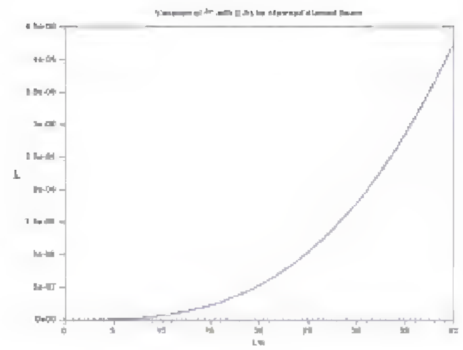

Figure 3: Variation of Buckling Load with L/h for Clamped-Clamed Beam

CONCLUSIONS

A consistent higher order theory for the analysis of sandwich beams having functionally graded faces has been developed here. The theory is formulated and assessed for different parameters. These parameters have significant influences on the mechanical behaviour of this type of structures. Since the face sheets made of FGM are suitable for the structures often subjected to the rapid variation of temperature, the buckling and post-buckling analysis is very important for analysis and design of this type of structures. Analytical solutions for the beam with functionally graded faces will be useful to assess the theory. This study will be of an important step in the development of functionally graded structures. The post-buckling behavior of a sandwich beam with functionally graded faces is investigated based on a higher-order shear deformation theory. The governing equations and the boundary conditions are derived using the Hamilton's principle and the governing equations are solved by closed-form method. The effects of length-to-thickness ratios and boundary conditions on buckling and post-buckling behaviors are analyzed in this study. From the analysis, it is observed that shear has the major effect on the critical buckling load and post-buckling response For the thin beam having high length-to-thickness ratios, the effect of shear deformation is negligible. In addition, the clamped-clamped beams take more axial load compare to the simply supported beams.

ACKNOWLEDGMENTS

The author is grateful to the Deanship of Scientific Research, Salman Bin Abdulaziz University, Al-Kharj, kingdom of Saudi Arabia for the financial support to this research study.

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